Hartree-Fock-Bogoliubov calculations of the isospin-symmetry-breaking correction in superallowed Fermi beta decay

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For accurate work, an isospin-symmetry breaking correction of between 0.5 % and 1.5 % is applied to ft values of nuclear beta decay. The correction is nucleus dependent and its evaluation dependent on nuclear-structure models. For convenience, the correction is divided into two components: $\delta_C = \delta_{C1} + \delta_{C2}$, where δ_{C1} arises from inserting charge-dependent terms into the Hamiltonian employed in a shell-model calculation, while δ_{C2} – the larger term – depends on radial-function differences between the proton and neutron involved in the beta transition. These radial functions are taken to be eigenfunctions of either a Saxon-Woods (SW) or a mean-field Hartree-Fock (HF) potential with adjustments to ensure that the radial functions have the appropriate asymptotic behavior.

To date, all HF calculations have been performed with the Skyrme parameterization of the nucleon-nucleon interaction. The computations assume spherical symmetry, which for open-shell nuclei is imposed by making the "filling approximation". Briefly, for partially filled orbitals a shell-model calculation is performed first to find the orbit occupancy. This occupancy is then imposed on the HF calculation and kept fixed during the HF iterations. An alternative strategy is to add a pairing term to the Skyrme interaction. At the end of each HF iteration, the standard pairing equations are solved to obtain the occupation probabilities, which are then returned to the HF code for the next iteration. This is known as a Hartree-Fock-Bogoliubov (HFB) procedure. In HF, the total energy of the nucleus is minimized subject to variations in the single-particle wave functions. In HFB, the total energy is minimized not only subject to variations in the single-particle wave functions, but also to variations in their occupation probabilities.

We take the form of the pairing interaction to be a delta function whose strength, V_0, is adjusted to reproduce the odd-even mass difference, E_α, where alpha denotes the valence orbital. However, a problem arose whenever the proton or neutron number corresponded to a closed sub-shell, because in this case the value of the pairing strength fell below a certain "critical value" and no superconducting solution could be found for the pairing equations. In this situation the occupation probability is constrained to be either 1 or 0, fully occupied or completely unoccupied. By contrast, the shell model produces solutions at sub-shell closures where the occupation probabilities are not just 1 or 0. To get such a result from a pairing model, we have to insist that the pairing strength be above the critical value. In such cases we fix the pairing strength, not to reproduce the odd-even mass difference, but to produce a "gap energy", Δ_α, of order 1 MeV. Fortunately, in the computation of the isospin-symmetry breaking correction, the dependence of δ_{C2} on the pairing strength is very weak, and a precisely determined value is not required.

Our procedure for obtaining the isospin-symmetry breaking correction, δ_{C2}, is essentially the one described in Ref. [1] except that HFB rather than HF calculations were performed. Results for one choice of the Skyrme interaction, SkM*, are given in Table I. It is seen that the addition of a pairing term has only a small impact on δ_{C2}. The average percentage shift between HF and HFB calculations was around 3%. In all cases, the shift is smaller than the error we assigned when we published our HF δ_{C2}

values in 2009 [1]. It is clear that no serious error has been made in our published work through the omission of pairing in the HF procedure.

Table I. Results of Hartree-Fock (HF) and Hartree-Fock-Bogoliubov (HFB) calculations with the Skyrme force, SkM*, for the isospin-symmetry-breaking correction, δ_{C2}. The columns headed by E_α contain the quasi-particle energies in MeV determined from odd-even mass differences; those headed by V_0 contain the adjusted strengths of the pairing delta function in MeV fm^3 units, and Δ_α are the resulting gap energies in MeV. Quantities marked with a dagger represent sub-shell closures.

Nucleus	α	E_α^p	E_α^n	V_0^p	V_0^n	Δ_α^p	Δ_α^n	δ_{C2}^{HFB}	δ_{C2}^{HF}	% Diff.
^{10}C	$p_{3/2}$	2.10	2.57	337	362	1.88	2.57	0.196	0.206	4.9
^{14}O	$p_{1/2}$	1.34	1.62	350	375†	1.35	1.03†	0.227	0.245	7.4
^{18}Ne	$d_{5/2}$	1.66	1.95	290	433†	1.21	0.98†	0.191	0.194	1.6
^{22}Mg	$d_{5/2}$	1.87	2.15	273	320	1.86	2.01	0.212	0.243	12.8
^{26}Si	$d_{5/2}$	1.62	1.88	247	288	1.25	1.78	0.341	0.335	1.8
^{30}S	$s_{1/2}$	0.83	1.07	272	325†	0.82	1.05†	0.521	0.536	2.8
^{34}Ar	$d_{3/2}$	1.19	1.39	305	350†	1.05	1.34†	0.492	0.498	1.2
^{38}Ca	$d_{3/2}$	1.35	1.53	289	311	1.20	1.53	0.604	0.598	1.0
^{42}Ti	$f_{7/2}$	1.34	1.56	237	335†	0.88	1.05†	0.546	0.538	1.5
^{26}Al	$d_{5/2}$	1.62	1.88	270	288	1.54	1.45	0.374	0.418	10.5
^{34}Cl	$d_{3/2}$	1.19	1.39	345†	303	1.00†	1.20	0.655	0.623	5.1
^{38}K	$d_{3/2}$	1.35	1.53	290	306	1.35	1.36	0.677	0.672	0.7
^{42}Sc	$f_{7/2}$	1.34	1.56	310†	279	1.06†	1.07	0.634	0.643	1.4
^{46}V	$f_{7/2}$	1.63	1.83	274	295	1.43	1.79	0.549	0.545	0.7
^{50}Mn	$f_{7/2}$	1.29	1.48	246	260	1.29	1.44	0.595	0.590	0.8
^{54}Co	$f_{7/2}$	1.13	1.35	230	245	0.98	0.90	0.647	0.649	0.3
^{62}Ga	$p_{3/2}$	1.13	1.33	286	290	1.13	1.24	0.951	0.942	1.0
^{66}As	$f_{5/2}$	1.15	1.68	300†	272	0.73†	0.98	1.135	1.122	1.2
^{70}Br	$f_{5/2}$	1.50	1.87	287	295	1.29	1.63	1.230	1.165	5.6
^{74}Rb	$f_{5/2}$	1.40	1.73	285	289	1.41	1.74	1.385	1.322	4.8

[1] J.C. Hardy and I.S. Towner, Phys. Rev. C **79**, 055502 (2009).